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A horizontal red brushstroke with a textured, painterly appearance, serving as a background for the text.

*Real numbers*

Part 7 (Rationalize)

## Definition

**Rationalizing the denominator means removing any radical term from the denominator of the fraction.**

**There are many types of rationalizing the denominator.**

**In this lesson we will learn how to rationalize the denominator of a fraction.**

# How to rationalize the denominator of a fraction?

❖ If the denominator is in form of  $a\sqrt{b}$  ;  $b > 0$ :

Multiply the numerator and the denominator by  $\sqrt{b}$  since  $\sqrt{b} \times \sqrt{b} = b$

Example 1:

$$\frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

# How to rationalize the denominator of a fraction?

❖ If the denominator is in form of  $a\sqrt{b}$  ;  $b > 0$ :

Multiply the numerator and the denominator by  $\sqrt{b}$  since  $\sqrt{b} \times \sqrt{b} = b$

Example 2:

$$\frac{3}{2\sqrt{7}}$$

$$\frac{3}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{2 \times 7} = \frac{3\sqrt{7}}{14}$$

# How to rationalize the denominator of a fraction?

❖ If the denominator is in form of  $a\sqrt{b}$  ;  $b > 0$ :

Multiply the numerator and the denominator by  $\sqrt{b}$  since  $\sqrt{b} \times \sqrt{b} = b$

Example 3:

$$\frac{3\sqrt{2}}{2\sqrt{18}}$$

Before rationalizing the denominator, it is preferable to simplify  $\sqrt{18}$

$$\frac{3\sqrt{2}}{2\sqrt{18}} = \frac{\cancel{3\sqrt{2}}}{2 \times \cancel{3\sqrt{2}}} = \frac{1}{2}$$



No need to  
rationalize

# How to rationalize the denominator of a fraction?

❖ If the denominator is in form of  $a \pm b\sqrt{c}$  ;  $c > 0$ :

Multiply the numerator and the denominator by a  $\mp b\sqrt{c}$  the conjugate of  $a \pm b\sqrt{c}$  according to the remarkable identity:

$$(x - y)(x + y) = x^2 - y^2$$

Example 1:

$$\frac{1}{1+\sqrt{3}}$$

$$\begin{aligned} \frac{1}{1+\sqrt{3}} &\times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{1-\sqrt{3}}{1^2 - \sqrt{3}^2} = \frac{1-\sqrt{3}}{1-3} = \frac{1-\sqrt{3}}{-2} \\ &= -\frac{1-\sqrt{3}}{2} \end{aligned}$$

# How to rationalize the denominator of a fraction?

Example 2:

$$\frac{3 + \sqrt{2}}{3 - \sqrt{2}}$$

$$\frac{3 + \sqrt{2}}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}} = \frac{(3 + \sqrt{2})^2}{3^2 - \sqrt{2}^2} = \frac{(3 + \sqrt{2})^2}{9 - 2} = \frac{(3 + \sqrt{2})^2}{7}$$

# How to rationalize the denominator of a fraction?

Example 3:

$$\frac{\sqrt{2}}{\sqrt{2} + 2\sqrt{3}}$$

$$\begin{aligned} & \frac{\sqrt{2}}{\sqrt{2} + 2\sqrt{3}} \times \frac{\sqrt{2} - 2\sqrt{3}}{\sqrt{2} - 2\sqrt{3}} = \frac{\sqrt{2}(\sqrt{2} - 2\sqrt{3})}{\sqrt{2}^2 - (2\sqrt{3})^2} \\ & = \frac{2 - 2\sqrt{6}}{4 - 12} = \frac{2 - 2\sqrt{6}}{-8} = \frac{2(1 - \sqrt{6})}{-8} = -\frac{1 - \sqrt{6}}{4} \end{aligned}$$

